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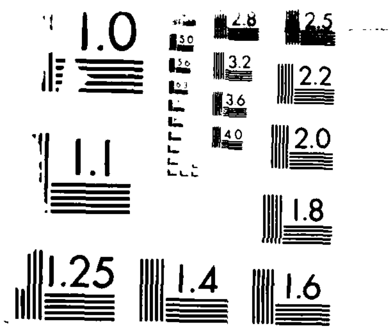
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Technical Report No. 2

October 15, 1986

Prepared Under Contract DAAG29-84-K-0191 with the U.S. Army Research Office

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Discretization Of A Semi-Markov Shadowing Process*

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ABSTRACT

A shadowing process in the plane is studied, with respect to a single source of light and a Poisson field of shadowing elements, located between the source of light and a target curve. N subsegments of length δ are considered within a specified portion of the target curve. Random measures of visibility and random shadow weights on the target curves are approximated by corresponding discrete random variables, defined for the N subsegments. Algorithms are provided for the computation of the distribution functions of the approximating discrete random variables.

Key words: Shadowing processes, Semi-Markov processes, discrete approximation, Poisson fields, measure of visibility.

*Research supported in Contract DAAG29-84-K-0191 with the Army Research Office.

1. *INTRODUCTION.*

Consider a *source of light* and a linear *path* in a plane. Random objects are scattered between the *source of light* and the *path*, casting shadows on portions of it.

The *Shadowing Model* was originally introduced by Chernoff and Dally [1]. In their paper they assumed that the shadowing objects are *disks* having centers which are distributed in a rectangle between the *source of light* and the *path*, according to a two dimensional *Poisson Process*, and their diameters are represented by i.i.d. random variables. The paper applies queueing theoretical techniques to derive the conditional distributions of the lengths of the shadowed and the unshadowed segments of the *path*.

The shadowed and unshadowed segments of the *path* are analogous to busy and idle periods in certain queueing processes with Markov arrivals. Like in these processes, the derivation of the distribution of the lengths of unshadowed segments (which are similar to idle periods) is straightforward. Chernoff's and Dally's paper concentrates on the distributions of shadowed segments, and provide some integral equations for these distributions. The solution of these equations requires generally lengthy numerical procedures.

Yadin and Zacks [2, 3] introduced the concept of *Random Measures of Visibility*, which are defined as the total length of the unshadowed subsegments of a specified segment of the *path*, or the proportion of the segment which is in the light. In these papers the distributions of *Measures of Visibility* were approximated by a mixture of a two points distribution on $\{0,1\}$ and a beta distribution on $(0,1)$. These *mixed beta* distributions were obtained by equating the first three moments of the distributions of *Measures of Visibility* to the first three moments of a mixed-beta distribution. In [2] the *path* was assumed to be a segment of a circle, centered at the origin at which the *source of light* was located. The centers of the disks were assumed to be distributed within an annular region in the circle. The results were extended in [3] and [4], which considered more general *paths* and *fields of obscuring objects*. The methodology was extended to a three dimensional model in [5]. Certain applications were discussed in [7].

The main objective of the present paper is to replace the *mixed beta* distributions by discrete distributions which approximate the required distributions. More specifically, we approximate the distribution of a measure of visibility, by the distribution of a discrete random variable, being the number of the proportion of the unshadowed subsegments, which are properly located on some segment of the *path.*

The shadowing model and the approximated *measures of visibility* and lengths of shadowed segments are formally defined in section 2. The approximating distributions can be given in terms of *probabilities of elementary events*, which are the events signifying which of the N subsegments are completely unshadowed among the set of all N subsegments. These probabilities can be obtained from *probabilities of simultaneous visibility*, namely the probabilities that all subsegments in a specified sets are unshadowed. Simultaneous visibility probabilities were determined for various cases in [4]. Appendix A presents the methodology needed for the derivations of *probabilities of simultaneous visibility.*

The accuracy of the discrete approximation to the distributions considered depends on the number of subsegments. Assuming a set of N subsegments, one has to compute 2^N *probabilities of elementary events* from the same number of *probabilities of simultaneous visibility.*

Any algorithm which is based on the *probabilities of simultaneous visibility* is necessarily non polynomial, thus is limited to a relatively small number of points, and may therefore yield insufficient accuracy. Fortunately, the distributions of the approximated *measures of visibility* can be obtained by a polynomial algorithm which is discussed in section 4.

The results of a numerical example are presented and discussed in section 5. These results illustrate the advantages and limitations of the algorithms developed in the present paper with respect to accuracy, computer time and memory requirements.

2. *The Shadowing Model*

Let C be a star-shaped rectifiable curve in the plane, with respect to a point $\underset{\sim}{O}$. C represents a possible path of a moving target and $\underset{\sim}{O}$ the location of an observer. We further assume that C does not pass through $\underset{\sim}{O}$. A countable number of objects are randomly dispersed between C and $\underset{\sim}{O}$. These random objects may obscure the visibility from $\underset{\sim}{O}$ of certain portions on C. The model assumes that the obscuring objects are disks centered according to a two-dimensional Poisson point process of rate λ, in a region S, located between $\underset{\sim}{O}$ and C and neither $\underset{\sim}{O}$ nor C are intersected by any such disk. Furthermore, it is assumed that the radii of these random disks are i.i.d. random variables, having a specified c.d.f., $F(x)$, concentrated on $[0, b]$.

Let Θ be an interval on the real line, representing the indices s of points $\underset{\sim}{P}_s \in C$ so that, if $s_1 < s_2$, $s_1, s_2 \in \Theta$, then $\underset{\sim}{P}_{s_1}$ is on the left of $\underset{\sim}{P}_{s_2}$. Let $\mathbf{R}_s$ designate a ray originating at $\underset{\sim}{O}$ and passing through $\underset{\sim}{P}_s$. We say that $\underset{\sim}{P}_s$ is visible from $\underset{\sim}{O}$ if $\mathbf{R}_s$ is not intersected by random disks centered in S. For each $s \in \Theta$, let $I(s)$ be a random variable assuming the value 1 if $\underset{\sim}{P}_s$ is visible from $\underset{\sim}{O}$, and the value 0 otherwise. The shadowing process on C is characterized by the indicator stochastic process

$$I = \{I(s) : s \in \Theta\}. \tag{2.1}$$

The Poisson field assumption implies that I is a semi-Markov process in which, $\{I(\tau) : \tau > s\}$ and $\{I(\tau) : \tau < s\}$ are conditionally independent, given $\{I(s) = 1\}$, for each $s \in \Theta$. Let $W(s)$ be a right-continuous non-decreasing function on Θ. For specified points $s', s'' \in \Theta$ such that $s' < s''$ define the random variable

$$V = \int_{s'}^{s''} I(s) dW(s). \tag{2.2}$$

V is called a *random visibility measure* on a segment $\bar{C}$, of C bounded by $\underset{\sim}{P}_{s'}$ and $\underset{\sim}{P}_{s''}$. The interpretation of V depends on the specific definition of the function W. For example, if $W(s)$ is the distance, measured along C between $\underset{\sim}{P}_{s'}$ and $\underset{\sim}{P}_s$, then V represents the total length of the portions of $\tilde{C}$, which are visible from $\underset{\sim}{O}$.

Another example is given when $W(s)$ is the time required for a target to move along C from $\underset{\sim}{P}_{s'}$ to $\underset{\sim}{P}_s$. In this case V is the total length of time in which the target is visible from $\underset{\sim}{O}$, while moving from $\underset{\sim}{P}_{s'}$ to $\underset{\sim}{P}_{s''}$. The weight of the shadow to the right of $\underset{\sim}{P}_s$ on C is defined as

$$X(s) = W(s + t(s)) - W(s), \tag{2.3}$$

where

$$t(s) = \inf\{\tau;\ \tau > 0,\ I(s+\tau) = 1\}. \tag{2.4}$$

Notice that $t(s)$ and $X(s)$ are random variables. Correspondingly, the shadowing *process* on C is the stochastic process

$$X = \{X(s) : s \in \Theta\}. \tag{2.5}$$

The Poisson field assumption implies that X is a Markov process, in which $\{X(\tau);\ \tau < s\}$ and $\{X(\tau);\ \tau > s\}$ are conditionally independent, given $X(s)$, for each $s \in \Theta$. Furthermore, all sample paths of X are piece-wise nonincreasing, having countable upwards jump points at which

$$\lim_{\epsilon \downarrow 0}(I(s) - I(s-\epsilon)) = -1. \tag{2.6}$$

These jump points are the left end limits of shadowed portions of C. In the following sections we provide discrete type approximations to the distribution of V and to the conditional distribution of $X(s)$, given that s satisfies (2.6).

3. *Discrete Approximations of V and X(s).*

Consider the segment $\tilde{C}$ with end points $\underset{\sim}{P}_{s'}$ and $\underset{\sim}{P}_{s''}$. Without loss of generality, assume that $W(s)$ is a c.d.f. concentrated on $[s', s'']$.

For a given positive integer N, let ξ_i be the (i/N)th fractile of W, i.e.,

$$\xi_i = W^{-1}(\frac{i}{N}) = \inf\{s : s \geq s' \text{ and } W(s) \geq \frac{i}{N}\},\ i = 0, \ldots, N. \tag{3.1}$$

Notice that $\xi_0 = s'$ and $\xi_N = s''$. These fractiles define a partition of $\tilde{C}$ to N subsegments. The measure of visibility V can be written, in terms of this partition as

$$V = \sum_{i=1}^{N} \int_{\xi_{i-1}}^{\xi_i} I(s)dW(s). \tag{3.2}$$

Let $\underset{\sim}{P}_{s_i}$ $(i = 1, \ldots, N)$ be the "midpoint" of the ith subsegment, defined by

$$s_i = W^{-1}(\frac{i - 1/2}{N}),\ i = 1, \ldots, N. \tag{3.3}$$

Let $J_{N,\delta}(s_i)(i = 1, \ldots, N)$ be a subsegment of weights δ/N, centered at $\underset{\sim}{P}_{s_i}$, $0 \leq \delta \leq 1$. These subsegments are specified by

$$J_{N,\delta}(s_i) = \{\underset{\sim}{P}_s : s_i^- \leq s < s_i^+\}, \tag{3.4}$$

where

$$s_i^{\pm} = W^{-1}(\frac{i - (1 \mp \delta)/2}{N}),\ i = 1, \ldots, N. \tag{3.5}$$

Finally, let $I_{N,\delta}(s_i)$ be an indicator random variable, which assumes the value 1 if $J_{N,\delta}(s_i)$ is completely visible.

The measure of visibility V can be approximated by replacing the function $I(s)$ in (3.2) with the indicators $I_{N,\delta}(s_i)$. Accordingly, the random variable V is approximated by a discrete random variable $V_{N,\delta}$, given by

$$\begin{aligned} V_{N,\delta} &= \sum_{i=1}^{N} \int_{\xi_{i-1}}^{\xi_i} I_{N,\delta}(s_i)dW(s) \\ &= \frac{1}{N} \sum_{i=1}^{N} I_{N,\delta}(s_i). \end{aligned} \tag{3.6}$$

This approximation converges uniformly to V (irrespective of δ), with probability one. Indeed,

$$
\begin{aligned}
(3.7) \qquad & |V_{N,\delta} - V| \le \frac{1}{N}\sum_{i=1}^{N} \sup\{|I_{N,\delta}(s_i) - \\
& I(s)|;\ \xi_{i-1} \le s < \xi_i\} \le \frac{2}{N} K(s', s''),
\end{aligned}
$$

where $K(s', s'')$ is the number of shadows on $\bar{C}$. The Poisson assumption implies that $K(s', s'')$ is finite with probability one, irrespective of N and δ.

Furthermore, for every realization, $I_{N,\delta}(s_i)$ is a decreasing function of δ. Hence, $V_{N,\delta}$ is a decreasing function of δ, converging to $V_{N,1}$ a.s. as $\delta \to 1$. In addition, $V_{N,1} \le V$ for each N, a.s.. This means that, for values of δ close to 1, the convergence of $V_{N,\delta}$ to V is from below.

The shadow-weights $X(s)$, for $\xi_{i-1} \le s < \xi_i$ are approximated by $X_{N,\delta}(s_i)$, $i = 1, \ldots, N$; which are discrete random variables defined as the proportion of subsegments $J_{N,\delta}(s_{i+j})$, on the right of $J_{N,\delta}(s_i)$, which are shadowed, i.e.

(3.8)

$$
X_{N,\delta}(s_i) = \begin{cases} 1 - \frac{i-1}{N}, & \text{if } I(s_{i+m}) = 0 \\ & \text{for all } m = 0, \ldots, N \\ \frac{1}{N}\inf\{m : 0 \le m \le N - i \text{ for which } I(s_{i+m}) = 1\}, & \text{otherwise.} \end{cases}
$$

Using the previous arguments about the nature of the Poisson process, we obtain that

$$
(3.9) \qquad \lim_{N\to\infty} \sup_{\xi_{i-1} \le s < \xi_i} |X_{N,\delta}(s_i) - X(s)| = 0, \text{ a.s.,}
$$

for each δ, $0 \le \delta \le 1$. Moreover, for each $N \ge 1$, and $i = 1, \ldots, N$, $\xi_{i-1} \le s < \xi_i$,

$$
(3.10) \qquad X(s) \le X_{N,1}(s_i), \text{ a.s..}
$$

We discuss now the approximations to the distributions of V and $X(s)$, given by the distributions of the discrete random variables $V_{N,\delta}$ and $\{X_{N,\delta}(s_i),\ i = 1, \ldots, N\}$.

Let $H(v)$ denote the c.d.f. of V. As shown in [4] and in Appendix A, the c.d.f. of V, under general conditions on the distributions of the radii of disks, has two jump points, at $v = 0$ and at $v = 1$, and is absolutely continuous on the open interval $(0,1)$. $p_0 = P\{V = 0\}$, is the probability that $\bar{C}$ is completely shadowed, and $p_1 = P\{V = 1\}$, is the probability that $\bar{C}$ is completely visible. Thus, generally

$$H(v) = \begin{cases} 0, & \text{if } v < 0 \\ p_0 + \int_0^v h(y)dy, & \text{if } 0 \le v < 1 \\ 1, & 1 \le v \end{cases} \tag{3.11}$$

where $h(y) \ge 0$, Riemann integrable, and

$$\int_0^1 h(y)dy = 1 - p_1 - p_0. \tag{3.12}$$

The c.d.f. of V, $H(v)$, is approximated by the c.d.f. of the discrete r.v. $V_{N,\delta}$. Let

$$h_{N,\delta}(k) = P\{V_{N,\delta} = \frac{k}{N}\},\ k = 0,1,\ldots,N. \tag{3.13}$$

The c.d.f. of $V_{N\delta}$, $H_{N,\delta}(x)$, is a step function, given by

$$H_{N,\delta}(x), = \begin{cases} 0, & \text{if } x < 0 \\ \sum_{j=0}^{k} h_{N,\delta}(j), & \text{if } \frac{k}{N} \le x < \frac{k+1}{N}\ (k = 0,1,\ldots,N-1) \\ 1, & \text{if } 1 \le x. \end{cases} \tag{3.14}$$

Notice that $h_{N,\delta}(0)$ is the discrete approximation to p_0 and $h_{N,\delta}(1)$ is an approximation to p_1. Notice that $h_{N,1}(1) = p_1$, and that $H_{N,1}(k) \ge H(\frac{k}{N})$, for each $N \ge 1$, $k = 0,1,\ldots,N$. Moreover, $p_0 \le h_{N,\delta}(0)$ for every $0 \le \delta \le 1$.

The distribution of $X(s)$ is concentrated on the interval $[0,\ 1 - W(s)]$. Typically, its c.d.f. has jump points at the two end points of the interval, and is absolutely continuous in its interior. We are interested in the conditional distribution of $X(s)$, given that $\underset{\sim}{P}_s$ is in a shadow, and the weight of the shadow to the left of $\underset{\sim}{P}_s$ is exactly y, for some $y > 0$.

Let $\bar{G}(x;s,y)$ denote the tail probability of this conditional distribution. Formally

$$\bar{G}(x;s,y) = P\{X(s) > x \mid I(\tau) = 0,\ \forall t_y(s) < \tau \leq s\}, \tag{3.15}$$

where

$$t_y(s) = \sup\{t : I(t) = 1 \text{ and } W(s) - W(t) \geq y\}. \tag{3.16}$$

The function $\bar{G}(x;s,y)$ can also be expressed in terms of the process I as

$$\begin{aligned} &\bar{G}(x;s,y) = P\{I(u) = 0 \textit{ for all} \\ &s < u < W^{-1}(W(s) + x) \mid I(\tau) = 0 \textit{ for all} \\ &t_y(s) < \tau \leq s\}. \end{aligned} \tag{3.17}$$

For each s in $[\xi_{i-1}, \xi_i)$ $(i = 1, \ldots, N)$ and $y = \frac{\nu}{N}$ $(\nu = 0, \ldots, i-1)$ we approximate $\bar{G}(x;s,y)$ by

$$\bar{G}_{N,\delta}(x;i,\nu) = \sum_j I\{j : j \geq N\ x + 1\} \cdot g_{N,\delta}(j;i,\nu), \tag{3.18}$$

where $g_{N,\delta}(j;i,\nu)$ is the conditional probability function

$$g_{N,\delta}(j;i,\nu) = P\{\prod_{l=i}^{i+j-1} (1 - I_l) I_{i+j} = 1 \mid I_{i-\nu-1} \prod_{l=i-\nu}^{i} (1 - I_l) = 1\}. \tag{3.19}$$

In the following sections we provide an algorithm for the computation of the probability functions $h_{N,\delta}(k)$ and $g_{N,\delta}(j;i,\nu)$.

4. *Recursive Determination Of* $h_{N,\delta}(k)$ *and* $g_{N,\delta}(j;i,\nu)$.

In the present section we develop a recursive algorithm for the determination of the probability functions under consideration, for specified values of N and δ. For the sake of simplicity, we replace $I_{N,\delta}(s_i)$ and $J_{N,\delta}(s_i)$ by I_i and J_i, respectively.

We start with definitions of probability functions, which play a role in the algorithm.

(i) For a given i, $i = 1,\ldots,N$, let $\xi_-(i)$ [resp. $\xi_+(i)$] denote the probability that the subsegment J_i is completely unshadowed by disks centered to the left [resp. to the right] of the ray $\mathbf{R}_{s_i}$.

(ii) For given indices $1 \le i < j \le N$, let $\xi(i,j)$ designate the probability that J_i and J_j are completely unshadowed by disks centered between the rays $\mathbf{R}_{s_i}$ and $\mathbf{R}_{s_j}$.

(iii) For $n = 0,\ldots,N-1$, $m = 1,\ldots,N-n$ and $k = 0,\ldots,m-1$, let $Q(n,m,k)$ denote the probability that J_n, J_{n+m}, and exactly k out of the $m-1$ subsegments in between, are completely unshadowed by disks centered between the rays $\mathbf{R}_{s_n}$ and $\mathbf{R}_{s_{n+m}}$.

For $n = 0$, $Q(0,m,k)$ is the probability that J_n, as well as exactly k of the $m-1$ subsegments to its left, are completely unshadowed by disks centered to the left of $\mathbf{R}_{s_m}$.

From the above definitions we obtain the following relationships:

$$Q(0,1,0) = \xi_-(1), \tag{4.1}$$

and

$$Q(n,1,0) = \xi(n,n+1),\ n = 1,\ldots,N-1. \tag{4.2}$$

Furthermore, the renewal nature of the processes implies the recursive formulae:

$$Q(n,m,k) = \sum_{\nu=k}^{m-1} Q(n,\nu,k-1)Q(n+\nu,m-\nu,0), \tag{4.3}$$

for $m = 2,\ldots,N-1$, $n = 0,\ldots,N-m$ and $k = 1,\ldots,m-1$. In addition, since

$$\sum_{k=0}^{m-1} Q(n,m,k) = \xi(n,n+m), \tag{4.4}$$

we obtain that

$$Q(n,m,0) = \xi(n, n+m) - \sum_{k=1}^{m-1} Q(n,m,k). \tag{4.5}$$

The functions $\xi_{\pm}(n)$ and $\xi(n,m)$ for special cases of two- and three-dimensional shadowing models, with $\delta = 0$, were previously presented in [4] and [5]. In Appendix A we present the general methodology of computing these functions.

The algorithm for computing the probability function $h_{N,\delta}(k)$, $k = 0, \ldots, N$, is given by the following recursive formulae:

Let $P_n(k)$, for $n = 1, \ldots, N$ and $k = 0, 1, \ldots, n$, denote the probabilities that k out of the first n subsegments, $J_1, \ldots, J_n$, are completely unshadowed. We compute these functions recursively, according to the formulae

$$P_1(k) = \begin{cases} Q(0,1,0)\xi_+(1), & k = 1 \\ \\ 1 - P_1(1), & \mathrm{k} = 0 \end{cases} \tag{4.6}$$

and for $n = 1, 2, \ldots, N$,

$$P_n(k) = \begin{cases} Q(0,n,n-1)\xi_+(n), & k = n \\ \\ P_{n-1}(k) + [Q(0,n,k-1) - Q(0,n,k)]\xi_+(n), & k = 1, \ldots, n-1 \\ \\ 1 - \sum_{j=1}^{n} P_n(k), & k = 0. \end{cases} \tag{4.7}$$

Finally,

$$h_{N,\delta}(k) = P_N(k), \; k = 0, 1, \ldots, N. \tag{4.8}$$

For the purpose of determining the conditional probability function $g_{N,\delta}(j; i, \nu)$ we introduce the function

$$q(n,m) = \begin{cases} Q(n,m,0), & \text{if } n + m \leq N \\ \\ \xi_+(n) - \sum_{\nu=1}^{N-m} q(n,\nu)\xi_+(n+\nu), & \text{if } n + m = N + 1. \end{cases} \tag{4.9}$$

When $n \geq 1$ and $n + m \leq N$ then the function $q(n,m)$ yields the probability that the subsegments J_n and J_{n+m} are completely unshadowed by disks centered between R_s and $\mathsf{R}_{s_{n+m}}$, while all the $(m-1)$ subsegments $\{J_{n+1}, \ldots, J_{n+m}\}$ are shadowed. $q(0,m)$ [resp. $q(N+1-m,m)$] is the probability that J_n [resp. J_{N+1-m}] is completely unshadowed by disks centered to the left [resp. to the right] of R_{s_m} [resp. $\mathsf{R}_{s_{N+1-m}}$], while all the subsegments to its left [resp. to its right] are shadowed.

Finally, the conditional probability function of $X_{N,\delta}$ is given by

$$(4.10) \qquad g_{N,\delta}(j;i,\nu) = \frac{q(i-v-1, j+\nu+1)\xi_+(i+j)\xi_-(i-v-1)}{\sum_{l=1}^{N-i+1} q(i-v-1, l+\nu+1)\xi_+(i+l)\xi_-(i-v-1)}$$

for $j = 1, \ldots, N-i+1$, where $\xi_+(N+1) \equiv 1$ and $\xi_-(0) \equiv 1$.

5. *A Special Case Of Linear Path And Trapezoidal Region.*

In the present section we illustrate the discrete approximations for a special geometrical configuration. We consider a shadowing process in the plane. The source of light is at the origin, $\underset{\sim}{O} = (0,0)$. The path curve $\mathcal{C}$ is a straight line perpendicular to the ray R_0, at distance R from $\underset{\sim}{O}$. The line segment $\tilde{\mathcal{C}}$ is bounded between points $\underset{\sim}{P}_{s'}$ and $\underset{\sim}{P}_{s''}$ on $\mathcal{C}$; where $\underset{\sim}{P}_s$ is a point on the line $\mathcal{C}$, having polar coordinates $(s, R/\cos(s))$.

The centers of disks are randomly dispersed, according to a two dimensional homogeneous Poisson process of intensity λ, between two lines, $\mathcal{U}$, $\mathcal{W}$, parallel to $\mathcal{C}$, at distances u and w, from $\underset{\sim}{O}$, respectively. We further assume that the radii of disks are independent random variables having a common uniform distribution on $[0,b]$. Disks which can cast shadows on $\tilde{\mathcal{C}}$ must have centers within the trapezoid bounded by $\mathcal{U}$, $\mathcal{W}$ and the rays $\mathrm{R}_{\theta'}$ and $\mathrm{R}_{\theta''}$, where

$$(5.1)\qquad \begin{aligned} \theta' &= \tan^{-1}(u\ \tan(s') - b/\cos(s')), \\ \theta'' &= tan^{-1}(u\ \tan(s'') + b/\cos(s'')). \end{aligned}$$

According to the general methodology for computing visibility probabilities (see Yadin and Zacks [3], [4]) we introduce the auxiliary functions $K_+(s,t)$ and $K_-(s,t)$, where $\lambda K_\pm(s,t)$ is the expected number of disks centered between the rays R_s and $\mathrm{R}_{s\pm t}$, which do *not* intersect R_s. As shown in detail in [7],

$$(5.2)\qquad K_-(s,t) = K_+(-s,t), \text{ for all } s' \le s \le s'',\ t > 0,$$

and

$$(5.3)\qquad K_+(s,t) = \begin{cases} K_+^{(1)}(s,t), & t \le \eta(\frac{b}{w}, s) \\ K_+^{(2)}(s,t), & \eta(\frac{b}{w}, s) < t \le \eta(\frac{b}{u}, s) \\ K_+^{(3)}(s,t), & \eta(\frac{b}{u}, s) < t, \end{cases}$$

where

$$(5.4)\qquad \eta(x,s) = \tan^{-1}(\tan(s) + \frac{x}{\cos(s)}) - s,$$

and

$$K_+^{(1)}(s,t) = \frac{w^3 - u^3}{6b} \cdot \frac{\sin^2(t)}{\cos(s)\ \cos^2(s+t)}, \tag{5.5}$$

$$\begin{aligned} K_+^{(2)}(s,t) &= K_+^{(1)}(s, \eta(\frac{b}{w}, s)) + \\ &\frac{w^2}{2}[\tan(s+t) - \tan(s)] + \frac{b^2}{6\cos(s)}[\frac{\cos(s+t)}{\sin(t)} - \\ &(\frac{u}{b})^3 \frac{\sin^2(t)}{\cos^2(s+t)} - 4\frac{w}{b} + \frac{u}{b}(\frac{u}{w})^2], \end{aligned} \tag{5.6}$$

$$K_+^{(3)}(s,t) = K_+^{(2)}(s, \eta(\frac{b}{u}, s)) + \frac{w^2 - u^2}{2}[\tan(t+s) - \tan(s)] - \frac{w^2 - u^2}{2} \cdot \frac{b}{u\cos(s)}. \tag{5.7}$$

Consider the weight function on $[s', s'']$

$$W(s) = \frac{R}{L}(\tan(s) - \tan(s')), \tag{5.8}$$

where $L = R(\tan(s'') - \tan(s'))$. $W(s)$ is the proportional distance along C of $\underset{\sim}{P}_s$ from $\underset{\sim}{P}_{s'}$, for $s' \leq s \leq s''$. Accordingly, the measure of visibility under consideration is the proportion of the total length of $\bar{C}$ which is visible, i.e.,

$$V = \frac{R}{L}\int_{s'}^{s''} I(s)\frac{ds}{\cos^2(s)}. \tag{5.9}$$

For a given N and δ, $0 \leq \delta \leq 1$, we approximate V by $V_{N,\delta}$, which is obtained by partitioning the line segment $\bar{C}$ to N equal-length subintervals. The points $\underset{\sim}{P}_{s_n}$ $(n = 1, \ldots, N)$ are the midpoints of these subintervals. The subsegments J_n $(n = 1, \ldots, N)$ are centered around $\underset{\sim}{P}_{s_n}$, with end-points $\underset{\sim}{P}_{s_n^-}$ and $\underset{\sim}{P}_{s_n^+}$ of distance $\delta/2N$ from $\underset{\sim}{P}_{s_n}$.

The functions $\xi_\pm(n)$ and $\xi(\nu, n)$, $0 < \nu < n \leq N$, defined in the previous sections are given, in terms of the K-functions, by the following formulae:

$$\xi_-(n) = \exp\{-\lambda[T(\theta', s_n) - K_-(s_n^-, s_n^- - \theta')]\}, \tag{5.10}$$

$$\xi_+(n) = \exp\{-\lambda[T(s_n, \theta'') - K_+(s_n^+, \theta'' - s_n^+)]\}, \tag{5.11}$$

and

$$\xi(\nu, n) = \exp\{-\lambda[T(s_\nu, s_n) - K_+(s^+, \frac{s_n^- - s_\nu^+}{2}) - K_-(s_N^-, \frac{s_n^- - s_\nu^+}{2})]\}, \tag{5.12}$$

where

$$T(\theta_1, \theta_2) = \frac{w^2 - u^2}{2}(\tan(\theta_2) - \tan(\theta_1)), \tag{5.13}$$

is the area of the trapezoid bounded by $\mathcal{U}$, $\mathcal{W}$, R_{θ_1} and R_{θ_2}.

6. *A Numerical Example.*

As in the previous section, we consider a linear curve with a trapezoidal region for disk centers. The radii disks are independent random variables having a uniform distribution on the interval $(0, 0.4)$. The path C is a horizontal straight line of distance $R = 1.0$ from the origin. The horizontal boundaries of the strip S are at distances $u = .4$ and $w = .6$, from $\underset{\sim}{O}$.

The segment $\bar{C}$ is the interval between $s' = -1$ and $s'' = 1$ on C. The intensity of the Poisson field is λ [disk centers/unit squared of area]. In Table 6.1 we present the c.d.f. of $V_{N,\delta}$, for $N = 40, 50$, $\delta = 0, 1.0$ and $\lambda = 10, 20$. The c.d.f. $H_{N,\delta}(x)$ is tabulated for $x = 0(0.1)1^-$. $H_{N,\delta}(0)$ is the discrete approximation to the value of p_0. $1 - H_{N,1}(1-)$ is the discrete approximation to the value of p_1. In addition, we provide in Table 6.1 the mixed-beta approximation to the c.d.f. $H(x)$. The mixed-beta c.d.f. is the function

$$
(6.1) \qquad H^*(x) = \begin{cases} \tilde{p}_0, & x = 0 \\ \tilde{p}_0 + (1 - \tilde{p}_0 - p_1) I_x(\alpha, \beta), & 0 < x < 1 \\ 1, & 1 \le x \end{cases}
$$

where $I_x(\alpha, \beta)$ is the incomplete beta-function ratio. The values of $\tilde{p}_0$, α and β are determined by equating the first three moments of V, which can be numerically determined (see Yadin and Zacks [3]), to the first three moments of $H^*(x)$. As seen in Table 6.1, $H_{50,1}(x) < H_{40,1}(x)$, for all $0 \le x < 1$. Indeed, $H_{N,1}(x)$ converges monotonically to $H(x)$ from above. Similarly, $H_{50,0}(0) < H_{40,0}(0)$ in all the two cases of $\lambda = 10$ and 20. Indeed, $H_{N,0}(0) \ge p_0$ for all N, and $H_{N,0}(0) \to p_0$ as $N \to \infty$. In Table 6.1 we see that, for each λ the values of the mixed-beta c.d.f. $H^*(x)$ satisfy

$$
H_{50,0}(x) < H^*(x) < H_{50,1}(x), \quad x = .2(.1).8.
$$

This indicates that the mixed-beta approximation $H^*(x)$ is apparently close to the true c.d.f. $H(x)$, over a wide range of x. In Table 6.2 we compare the moments of $H_{N,0}(x)$,

$H_{N,1}(x)$ and $H^*(x)$. We see that the moments of $H_{N,0}(x)$ are very close to those of $H^*(x)$. Recalling that the first three moments of $H^*(x)$ are, by definition, the corresponding moments of V, we conclude that the moments of $H_{N,0}(x)$ are close to the true ones.

Table 6.1 *Values of* $H_{N,\delta}(x)$ *and* $H^*(x)$ *for*

$R = 1.,\ u = .4,\ w = .6,\ s' = -1,\ s'' = 1,\ \lambda = 10, 20,\ b = .4.$

		$\delta = 0$		$\delta = 1$		
λ	x	$N = 40$	$N = 50$	$N = 40$	$N = 50$	$H^*(x)$
	0.0	0.00081	0.00072	0.00225	0.00167	0.0000
10	0.1	0.00461	0.00501	0.01032	0.00962	0.0021
	0.2	0.02325	0.02461	0.04286	0.04036	0.0305
	0.3	0.07009	0.07315	0.11159	0.10657	0.0942
	0.4	0.15805	0.16336	0.22352	0.21624	0.1967
	0.5	0.29110	0.29864	0.37336	0.36519	0.3339
	0.6	0.45836	0.46724	0.54266	0.53547	0.4954
	0.7	0.63657	0.64548	0.70731	0.70265	0.6645
	0.8	0.79584	0.80301	0.84177	0.83984	0.8190
	0.9	0.90577	0.91018	0.92848	0.92824	0.9325
	p_1	0.02620	0.02520	0.02140	0.02140	0.0214
	0.0	0.01588	0.01448	0.03489	0.02796	0.0084
20	0.1	0.06019	0.06440	0.10746	0.10287	0.0787
	0.2	0.18754	0.19516	0.27790	0.26839	0.2372
	0.3	0.36854	0.37822	0.47959	0.46829	0.4278
	0.4	0.56755	0.57417	0.66784	0.65805	0.6121
	0.5	0.73678	0.74463	0.81370	0.80708	0.7668
	0.6	0.86270	0.[illegible]6802	0.90931	0.90579	0.8803
	0.7	0.93964	0.94263	0.96245	0.96102	0.9515
	0.8	0.97839	0.97974	0.98712	0.98672	0.9869
	0.9	0.99400	0.99449	0.99651	0.99646	0.9983
	p_1	0.00069	0.00054	0.00046	0.00046	0.0005

Table 6.2 *Moments of* $H_{N,\delta}(x)$ *and of* $H^*(x)$ *for*

$R = 1.,\ u = .4,\ w = .6,\ s' = -1,\ s'' = 1,\ \lambda = 10, 20,\ b = .4.$

		$\delta = 0$		$\delta = 1$		
	n	$N = 40$	$N = 50$	$N = 40$	$N = 50$	$H^*(x)$
$\lambda = 10$	1	0.5933	0.5933	0.5488	0.5574	0.5933
	2	0.3951	0.3950	0.3477	0.3565	0.3949
	3	0.2843	0.2842	0.2416	0.2493	0.2840
	4	0.2167	0.2165	0.1794	0.1860	0.2158
	5	0.1726	0.1725	0.1402	0.1458	0.1710
	6	0.1425	0.1423	0.1140	0.1188	0.1400
	7	0.1209	0.1207	0.0957	0.0999	0.1177
	8	0.1051	0.1048	0.0825	0.0861	0.1011
	9	0.0930	0.0928	0.0726	0.0758	0.0884
	10	0.0837	0.0834	0.0650	0.0679	0.0785
	1	0.3556	0.3556	0.3043	0.3139	0.3556
$\lambda = 20$	2	0.1031	0.1630	0.1278	0.1340	0.1629
	3	0.0868	0.0867	0.0638	0.0677	0.0865
	4	0.0513	0.0512	0.0360	0.0385	0.0508
	5	0.0329	0.0329	0.0222	0.0239	0.0321
	6	0.0225	0.0225	0.0148	0.0160	0.0215
	7	0.0162	0.0162	0.0104	0.0113	0.0151
	8	0.0122	0.0122	0.0077	0.0084	0.0109
	9	0.0095	0.0095	0.0059	0.0064	0.0082
	10	0.0077	0.0076	0.0047	0.0051	0.0063

We conclude this section with a numerical example of the conditional distribution of the remaining shadow length, given that ν subsegments to the left of the ith subsegment are in shadow. In Tables 6.3 and 6.4 we present the conditional p.d.f. (4.10), for the case of a trapezoidal region, with $R = 1$, $u = .4$, $w = .6$, $s' = -1$, $s'' = 1$, $b = .2$ and $\lambda = 25, 50$. The interval $[s', s'']$ is partitioned into $N = 20$ subintervals. Table 6.3 presents these distributions for $\delta = 0$ and Table 6.4 for $\delta = 1$. For the sake of space saving, we present these conditional PDF's for $i = 10$; $j = 1, 2, \ldots, 11$ and $\nu = 0, 1, 2$ and 3 only. Since $N = 20$, $j = 11$ corresponds to the case that *all* the 10 subsegments to the right of J_{10} are shadowed. We see that when $\delta = 0$, the modes of all the conditional distributions are at $j = 1$; which means that it is most likely that the eleventh subsegment, J_{11}, is in the light. This is not necessarily the case when $\delta = 1$.

Table 6.3 *Values of the Conditional P.D.F.* $g(j; i, \nu)$, *for* $i = 10$, *in a discrete approximation, with* $N = 20$, $\delta = 0$.

λ	ν	j	g	ν	j	g	ν	j	g	ν	j	g
25	0	1	0.3296	1	1	0.4649	2	1	0.4817	3	1	0.4573
		2	0.3065		2	0.2507		2	0.2323		2	0.2479
		3	0.1634		3	0.1221		3	0.1251		3	0.1287
		4	0.0808		4	0.0667		4	0.0660		4	0.0680
		5	0.0456		5	0.0363		5	0.0360		5	0.0372
		6	0.0258		6	0.0206		6	0.0205		6	0.0211
		7	0.0152		7	0.0122		7	0.0121		7	0.0125
		8	0.0094		8	0.0075		8	0.0075		8	0.0078
		9	0.0061		9	0.0048		9	0.0048		9	0.0050
		10	0.0040		10	0.0032		10	0.0032		10	0.0033
		11	0.0136		11	0.0109		11	0.0108		11	0.0112
50	0	1	0.2208	1	1	0.2836	2	1	0.2919	3	1	0.2845
		2	0.2146		2	0.2016		2	0.1949		2	0.1982
		3	0.1490		3	0.1329		3	0.1333		3	0.1343
		4	0.0979		4	0.0906		4	0.0900		4	0.0907
		5	0.0673		5	0.0617		5	0.0613		5	0.0619
		6	0.0466		6	0.0427		6	0.0425		6	0.0429
		7	0.0329		7	0.0302		7	0.0300		7	0.0303
		8	0.0238		8	0.0218		8	0.0217		8	0.0219
		9	0.0175		9	0.0161		9	0.0150		9	0.0161
		10	0.0132		10	0.0121		10	0.0120		10	0.0121
		11	0.1164		11	0.1068		11	0.1062		11	0.1071

Table 6.4. *Values of The Conditional P.D.F.* $g_{N,\delta}(j; i, \nu)$, *for* $i = 10$, in a discrete approximation, with $N = 20$, $\delta = 1$.

λ	ν	j	g	ν	j	g	ν	j	g	ν	j	g
25	0	1	0.1136	1	1	0.2161	2	1	0.2817	3	1	0.2741
		2	0.1885		2	0.2183		2	0.1904		2	0.1839
		3	0.1868		3	0.1434		3	0.1287		3	0.1352
		4	0.1230		4	0.0972		4	0.0941		4	0.0959
		5	0.0831		5	0.0714		5	0.0670		5	0.0681
		6	0.0617		6	0.0514		6	0.0481		6	0.0491
		7	0.0452		7	0.0374		7	0.0352		7	0.0359
		8	0.0334		8	0.0278		8	0.0261		8	0.0266
		9	0.0252		9	0.0210		9	0.0197		9	0.0201
		10	0.0193		10	0.0161		10	0.0151		10	0.0154
		11	0.1200		11	0.0998		11	0.0938		11	0.0957

Appendix A. Visibility Probabilities For Poisson Fields.

We present here the basic methodology of determining visibility probabilities, under Poisson shadowing processes, leading to general formulae of $\xi_{\pm}(n)$ and $\xi(n,m)$. For more details and discussion see Yadin and Zacks [2], [3], [4].

We assume that a countable number of disks are randomly dispersed in a region in the plane according to a Poisson random field model. More specifically, let (θ,ρ) be the polar coordinates of the location of a center of a disk, with respect to an origin $\underset{\sim}{O}$, and let r be the radius of a disk. Let

$$(A.1) \qquad \mathcal{S} = \{(\theta,\rho,r) : (\theta,\rho,r) \in S\},$$

be the sample space of possible disks, and let $\mathcal{T}$ be the Borel σ-field of subsets of $\mathcal{S}$. Let $T \in \mathcal{T}$ and $N\{T\}$ be the number of disks having coordinates in T. The Poisson field model assumes that, for any $t \geq 1$ and any partition $\{T_1,\ldots,T_t\}$ of $\mathcal{S}$, $N\{T_1\},\ldots,N\{T_t\}$ are independent random variables, having Poisson distributions with means

$$(A.2) \qquad \nu\{T_i\} = \lambda \iiint_{T_i} h(\theta,\rho)dF(r \mid \theta,\rho)d\theta d\rho, \; i = 1,\ldots,t,$$

where $0 < \lambda < \infty$ is the intensity of the field; $F(r \mid \theta,\rho)$ a conditional c.d.f. of r, given (θ,ρ) and $h(\theta,\rho)$ is a bivariate density function on $(-\pi,\pi] \times (0,\infty)$.

Let $\underset{\sim}{P} = (s,\rho)$ be a point in the plane, and let $\mathrm{R}(s)$ be a ray from $\underset{\sim}{O}$, passing through $\underset{\sim}{P}$. Let $U_-(s)$ [resp. $U_+(s)$] be all points in $\mathcal{S}$, on the left [resp. right] of $\mathrm{R}(s)$, which intersect $\mathrm{R}(s)$, i.e., if $d(\theta,\rho;\mathrm{R}(s))$ denotes the distance between $\underset{\sim}{P} = (\theta,\rho)$ and $\mathrm{R}(s)$ then,

$$(A.3) \qquad U_-(s) = \{(\theta,\rho,r) : (\theta,\rho,r) \in S,\; \theta < s, \text{ and } d(\theta,\rho;\mathrm{R}(s)) < r\}.$$

Similarly,

$$(A.4) \qquad U_+(s) = \{(\theta,\rho,r) : (\theta,\rho,r) \in S,\; \theta > s, \text{ and } d(\theta,\rho;\mathrm{R}(s)) < r\}.$$

Let $\lambda M_+(s)$ [resp. $\lambda M_-(s)$] be the expected member of disk on the right [resp. left] of $\mathsf{R}(s)$ which intersect it, i.e.,

$$(A.5) \qquad M_\pm(s) = \iiint\limits_{U_\pm(s)} h(\theta,\rho)dF(r \mid \theta,\rho)]d\theta d\rho.$$

The probability that $\mathsf{R}(s)$ is *not* intersected by disks on its left [resp. right] is $\tilde{\xi}_-(s)$ [resp. $\tilde{\xi}_+(s)$], where

$$(A.6) \qquad \tilde{\xi}_\pm(s) = \exp\{-\lambda M_\pm(s)\}.$$

The *visibility probability* of $\underset{\sim}{P} = (s,\rho)$ is $\tilde{\xi}(s) = \tilde{\xi}_+(s)\tilde{\xi}_-(s)$.

If $\bar{C}$ is a *segment of a* curve in the plane, and $\underset{\sim}{P}_n = (s_n,\rho_n)$, $n = 1,\ldots,N$, are "mid-points" in a partition of $\bar{C}$ into N subsegments, then, in the notation of the previous sections $\xi_\pm(n) = \tilde{\xi}_\pm(s_n)$, $n = 1,\ldots,N$.

Let $\underset{\sim}{P}_1 = (s_1,\rho_1)$ and $\underset{\sim}{P}_2 = (s_2,\rho_2)$ are two distinct points and $\mathsf{R}(s_i)$, $i = 1,2$ are the corresponding rays from the $\underset{\sim}{O}$ through the points. Let $U(s_1,s_2)$ be the set of all possible disks in S, between $\mathsf{R}(s_1)$ and $\mathsf{R}(s_2)$ which intersect either $\mathsf{R}(s_1)$ or $\mathsf{R}(s_2)$, i.e.,

$$(A.7) \qquad U(s_1,s_2) = \{(\theta,\rho,r) : (\theta,\rho,r) \in S \text{ and } d(\theta,\rho;\mathsf{R}(s_1)) < r \text{ or } d(\theta,\rho;\mathsf{R}(s_2)) < r\}.$$

The expected number of disks between $\mathsf{R}(s_1)$ and $\mathsf{R}(s_2)$ which intersect either one of these rays is $\lambda M(s_1,s_2)$, where

$$(A.8) \qquad M(s_1,s_2) = \iiint\limits_{U(s_1,s_2)} h(\theta,\rho)dF(r \mid \theta,\rho)d\theta d\rho.$$

Finally, the probability that neither $\mathsf{R}(s_1)$ nor $\mathsf{R}(s_2)$ are intersected by disks centered between them is $\tilde{\xi}(s_1,s_2) = \exp\{-\lambda M(s_1,s_2)\}$. The value of the function $\xi(n_1,n_2)$ for points P_1 and P_2 on $\bar{C}$ is given by $\xi(n_1,n_2) = \tilde{\xi}(s_1,s_2)$.

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: Technical Report No. 2	2. GOVT ACCESSION NO.: AD-A175 399
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): Discretization of a Semi-Markov Shadowing Process	5. TYPE OF REPORT & PERIOD COVERED: Technical
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): S. Zacks and M. Yadin	8. CONTRACT OR GRANT NUMBER(s): DAAG29-84-K-0191
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Center for Statistics, Quality Control & Design, State University of New York, University Center At Binghamton, Binghamton, NY 13901	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: U. S. Army Research Office, Post Office Box 12211, Research Triangle Park, NC 27709	12. REPORT DATE: October 15, 1986
	13. NUMBER OF PAGES: 25
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

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Shadowing processes, Semi-Markov processes, discrete approximation, Poisson Fields, measure of visibility

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A shadowing process in the plane is studied, with respect to a single source of light and a Poisson field of shadowing elements, located between the source of light and a target curve. N subsegments of length δ are considered within a specified portion of the target curve. Random measures of visibility and random shadow weights on the target curves are approximated by corresponding discrete random variables, defined for the N subsegments. Algorithms are provided for the computation of the distribution functions of the approximating discrete random variables.

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